

解 答

2025	科目名	数学（解析）
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$$\begin{aligned}
 (1) \quad f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0, y_0) &= f_x(x_0, y_0)\Delta x + f_y(x_0, y_0)\Delta y \\
 &\quad + \frac{1}{2}\{f_{xx}(x_0 + \theta\Delta x, y_0 + \theta\Delta y)(\Delta x)^2 + 2f_{xy}(x_0 + \theta\Delta x, y_0 + \theta\Delta y)\Delta x\Delta y \\
 &\quad + f_{yy}(x_0 + \theta\Delta x, y_0 + \theta\Delta y)(\Delta y)^2\}
 \end{aligned}$$

ここで $0 < \theta < 1$.

$$(2) \quad f_x(x_0, y_0) = f_y(x_0, y_0) = 0.$$

$$(3) \quad f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0, y_0) = \frac{1}{2} \vec{d}^T \begin{pmatrix} f_{xx}(x_0 + \theta\Delta x, y_0 + \theta\Delta y) & f_{xy}(x_0 + \theta\Delta x, y_0 + \theta\Delta y) \\ f_{xy}(x_0 + \theta\Delta x, y_0 + \theta\Delta y) & f_{yy}(x_0 + \theta\Delta x, y_0 + \theta\Delta y) \end{pmatrix} \vec{d}$$

(4) 十分条件は

$$f_{xx}(x_0, y_0) > 0, \quad \begin{vmatrix} f_{xx}(x_0, y_0) & f_{xy}(x_0, y_0) \\ f_{xy}(x_0, y_0) & f_{yy}(x_0, y_0) \end{vmatrix} > 0$$

（理由）2次導関数が連続なので、上の条件が成り立つとき、十分小さな $|\Delta x|, |\Delta y|$ に対して、

$$f_{xx}(x_0 + \Delta x, y_0 + \Delta y) > 0, \quad \begin{vmatrix} f_{xx}(x_0 + \theta\Delta x, y_0 + \theta\Delta y) & f_{xy}(x_0 + \theta\Delta x, y_0 + \theta\Delta y) \\ f_{xy}(x_0 + \theta\Delta x, y_0 + \theta\Delta y) & f_{yy}(x_0 + \theta\Delta x, y_0 + \theta\Delta y) \end{vmatrix} > 0$$

となるので.

$$(5) \quad (i) \quad f_x = 2e^{-x^2-y^2}x(-\alpha x^2 - \beta y^2 + \alpha), f_{xx} = -2xf_x + 2e^{-x^2-y^2}(-3\alpha x^2 - \beta y^2 + \alpha)$$

$$(ii) \quad \text{極小値 } f(0,0) = 0. \quad \text{極大値 } f(0, \pm 1) = e^{-1}\beta$$